Bellwork: 2/26/13
List all the factors of...

1) 48

1	48
2	24
3	16
4	12
6	8

2) 81

1	81
3	27
9	9

3) 72

1	72
2	36
3	24
4	18
6	12
8	9

ALGEBRA 2 NAME ________________________
NOTES 5.5 DATE: _______________PD________

THE RATIONAL ZERO TEST - FINDING ACTUAL ZEROS

The following test will allow you to determine ***which*** numbers to consider as possible roots.

> ***RATIONAL ZERO TEST:***
> If polynomial $f(x)$ has **integer** coefficients, then every ***rational zero*** of this polynomial has the form:
>
> $\dfrac{p}{q}$ where $\begin{cases} p \text{ is a factor of the constant term of } f(x) \text{ and} \\ q \text{ is a factor of the leading coefficient of } f(x) \end{cases}$

Example 1: Find the ***possible rational roots (zeros)*** of $f(x) = 2x^3 + 3x^2 - 8x + 3$.

Constant term is 3 ➔ Factors: $\pm1, \pm3$ ➔ p

Leading coefficient ➔ Factors: $\pm1, \pm2$ ➔ q

$$\frac{1, 3}{1, 2}$$

Possible rational roots: $\dfrac{p}{q} = \dfrac{\pm1, \pm3}{\pm1, \pm2} = \pm1, \pm3, \pm\dfrac{1}{2}, \pm\dfrac{3}{2}$

There are 8 *possible rational* roots, however we know that this polynomial has *at most* 3 roots. This means that no more than 3 of these numbers will work.

Use synthetic division to test the possible rational roots to ***find the actual zeros***:

-6

$$\begin{array}{r|rrrr} 1 & 2 & 3 & -8 & 3 \\ & & 2 & 5 & -3 \\ \hline & 2 & 5 & -3 & \boxed{0} \end{array}$$

➔ $f(x) = (x-1)(2x^2 + 5x - 3)$
$= (x-1)(2x-1)(x+3)$

$(2x^2 + 6x)(-1x - 3)$
$2x(x+3) - 1(x+3)$

Actual Roots: $1, \frac{1}{2}, -3$

$x - 1 = 0$ $\quad 2x - 1 = 0$ $\quad x + 3 = 0$
$x = 1$ $\quad 2x = 1$ $\quad x = -3$
$x = \frac{1}{2}$

Example 2: Find the actual zeros of $f(x) = 3x^3 + 4x^2 - 13x + 6$.

Maximum # of roots: 3

$\frac{1, 2, 3, 6}{1, 3}$ Possible rational roots: $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{3}, \pm \frac{2}{3}$

Factored form: $(x-1)(x+3)(3x-2)$

Actual roots: $1, -3, \frac{2}{3}$

Work:

$$\begin{array}{r|rrrr} 1 & 3 & 4 & -13 & 6 \\ + & \downarrow & 3 & 7 & -6 \\ \hline & 3 & 7 & -6 & \boxed{0} \end{array}$$

$3x^2 + 7x - 6$ -18

$(3x^2 + 9x)(-2x - 6)$

$3x(x+3) - 2(x+3)$

$(x+3)(3x-2)$

$x+3=0$, $x=-3$

$3x-2=0$, $3x=2$, $x=\frac{2}{3}$

Note: With a 3rd degree polynomial, you only need to find 1 root using synthetic division. Then factor or use quadratic formula to find remaining roots of the polynomial.

Example 3: Find the actual zeros of $f(x) = -2x^4 + 13x^3 - 21x^2 + 2x + 8$.

Maximum # of roots: 4

$\frac{1, 2, 4, 8}{1, 2}$ $\frac{\text{last \#}}{\text{first \#}}$ Possible rational roots: $\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}$

Factored form: $-1(x-1)(x-2)(2x+1)(x-4)$

Actual roots: $1, 2, 4, -\frac{1}{2}$

Work:

$$\begin{array}{r|rrrrr} 1 & -2 & 13 & -21 & 2 & 8 \\ + & \downarrow & -2 & 11 & -10 & -8 \\ \hline 2 & -2 & 11 & -10 & -8 & \boxed{0} \\ + & \downarrow & -4 & 14 & 8 & \\ \hline & -2 & 7 & 4 & \boxed{0} & \end{array}$$

$-2x^2 + 7x + 4$ -8

$-1(2x^2 - 7x - 4)$

$(2x^2 + x)(-8x - 4)$

$x(2x+1) - 4(2x+1)$

$-1(x-4)(2x+1)$

$x-4=0$, $x=4$

$2x+1=0$, $2x=-1$, $x=-\frac{1}{2}$

Homework: pg 316 #9-17

just find the possible zeros